

Looking for bound states and resonances in the $\eta' K \bar{K}$ system

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Motivated by the continuous experimental investigations of $X(1835)$ in three-body decay channels like $\eta' \pi^+ \pi^-$, we investigate the $\eta' K \bar{K}$ system with the aim of searching for bound states and/or resonances when the dynamics involved in the $K \bar{K}$ subsystem can form the resonances: $f_0(980)$ in isospin 0 or $a_0(980)$ in isospin 1. For this, we solve the Faddeev equations for the three-body system. The input two-body t -matrices are obtained by solving Bethe-Salpeter equations in a coupled channel formalism. As a result, no signal of a three-body bound state or resonance is found.

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An observation of a resonance-like structure around 1830 MeV, $X(1835)$, has been reported in several processes, with the recent-most finding being in the mass spectrum of $\eta' \pi^+ \pi^-$ by the BES collaboration [1]. The first observation of $X(1835)$ in the $\eta' \pi^+ \pi^-$ mass spectrum, in the process $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$, was discussed in Ref. [2], where a Breit-Wigner fit to the data yielded a mass $M = 1833.7 \pm 6.1 \pm 2.7$ MeV a width $\Gamma = 67.7 \pm 20.3 \pm 7.7$ MeV. The same process is studied with a larger statistics by BESIII in Ref. [1] where, apart from the confirmation of $X(1835)$, the finding of two new states is reported: $X(2120)$ and $X(2370)$. A more recent analysis of the $\eta' \pi^+ \pi^-$ data [3], focussed on the energy region of $X(1835)$, shows that a fit to the data in this region requires either the presence of a much broader state ($\Gamma \sim 247$ MeV), distorted by the cusp of $p\bar{p}$, or an interference between a broad and a narrow state. The fit shows that the broad state, in any case, couples strongly to the $p\bar{p}$ system [3]. An enhancement near the $p\bar{p}$ threshold in the BES data has been found in some processes (like $J/\psi \rightarrow \gamma p\bar{p}$, $\psi(2s) \rightarrow \gamma p\bar{p}$ [4]) but not in some other processes (like, $J/\psi \rightarrow \omega p\bar{p}$ [5], $J/\psi \rightarrow \phi p\bar{p}$ [6]). The decay of $\psi(2s)$ has been studied by the CLEO Collaboration also, but the data shows no $p\bar{p}$ threshold enhancements in the mass spectra of $\gamma p\bar{p}$, $\pi^0 p\bar{p}$ and $\eta p\bar{p}$ [7]. All these findings have generated a series of discussions on the possibility of the existence of a baryonium, or other alternative explanations of the enhancement seen around 1830 MeV [8–19]. A resonance like structure around 1830 MeV is also found in the mass spectrum of $\eta K \bar{K}$ [20], $\eta \pi^+ \pi^-$, where the $K \bar{K}$ is found to come dominantly from $f_0(980)$ in the former case. It is not clear if all the states found around 1830 MeV in different systems are the same and the origin of this/these state(s) is still an open question. In the present manuscript we study the possibility of understanding $X(1835)$ as a bound state arising from three pseudoscalar dynamics involving the η' -meson.

The dynamics of a system of pseudoscalar mesons is related to the low energy regime of the Quantum Chromodynamics (QCD), which can be described in terms of the chiral perturbation theory (χ PT). The latter is an effective field theory based on the fact that the QCD Lagrangian with massless u , d and s quarks has an $SU(3)_R \times SU(3)_L$ chiral symmetry. This

symmetry is spontaneously broken to $SU(3)_V$, giving rise to an octet of Goldstone bosons, which are identified with the octet formed by the pseudoscalar mesons: π , K and η . Particles which become massless in the chiral limit of zero quark masses, $m_{u,d,s} \rightarrow 0$. The ninth pseudoscalar, the η' meson, which was found independently, but almost at the same time, by two collaborations [21, 22] in 1964, is an interesting hadron: it is closely related to the axial $U_A(1)$ anomaly [23–25]. This fact prevents the η' -meson to become massless even in the chiral limit. Thus, the η' -meson is not included explicitly in the Lagrangian in the conventional χ PT.

A way to incorporate η' , however, could be inspired by the works of Witten, 't Hooft and others [24, 25], who showed that in the limit of infinite number of colors ($N_c \rightarrow \infty$) of QCD the $SU(3)$ singlet state, η_1 , is massless and the global $SU(3)_R \times SU(3)_L$ symmetry is replaced by $U(3)_R \times U(3)_L$. This is because in the large N_c limit the anomaly related to the axial current is $1/N_c$ suppressed. This fact can be used to incorporate η' in an effective field theory based on chiral symmetry, since η_1 becomes the ninth Goldstone boson and can be included in an extended $U(3)_R \times U(3)_L$ chiral Lagrangian (see, for example, [26–28] for more details). Alternative approaches to include the singlet state in an effective field theory have also been developed [29, 30].

Thus, to build a Lagrangian based on chiral symmetry and including at the same time the η' meson, in the spirit of Refs. [24–30], the physical η and η' fields are introduced as the admixtures of the $SU(3)$ singlet η_1 and octet η_8 states. Indeed, the $\eta - \eta'$ mixing has received a lot of attention in the recent past. Usually, within the mixing scheme, the η and η' mesons are considered as linear combinations of η_1 and η_8 through a mixing angle θ

$$\begin{aligned} |\eta\rangle &= \cos\theta |\eta_8\rangle - \sin\theta |\eta_1\rangle, \\ |\eta'\rangle &= \sin\theta |\eta_8\rangle + \cos\theta |\eta_1\rangle. \end{aligned} \quad (1)$$

The values obtained for this mixing angle range, typically, from -13° to -22° . These values are extracted, just to mention a few examples, from the decays of η and η' to two photons, decays of J/ψ , etc. [31–34]. Considering this mixing angle, the $SU(3)$ matrix containing the Goldstone bosons can be extended to $U(3)$ as

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \frac{2}{\sqrt{6}}\eta' \end{pmatrix}, \quad (2)$$

where the standard $\eta - \eta'$ mixing is considered [Eq. (1) with $\sin\theta = -1/3$, thus $\theta \sim -20^\circ$]. Also, a two-mixing angle scheme has been proposed [36, 37] and adopted to explain some decay widths of the η and η' mesons, radiative decays, pseudoscalar decay constants, and other quantities [38, 39]. We stick to the approach with one mixing angle.

Using the matrix in Eq. (2), at leading order in large N_c , the lowest order Lagrangian describing the interaction between two pseudoscalar mesons is given by [26–28, 35]

$$\mathcal{L} = \frac{1}{12f^2} \langle (\partial_\mu \phi \phi - \phi \partial_\mu \phi)^2 + M \phi^4 \rangle, \quad (3)$$

with $M = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$.

The interaction of the η' meson with other pseudoscalars in S-wave is rather weak, and neither bound state nor resonance have been found theoretically due to this dynamics. However, it was shown in Refs. [28, 35] that inclusion of η' in the coupled channel analysis is required to reproduce the isospin $I = 1/2$ and $I = 3/2$ S-wave $K\pi$ phase-shift up to energies of 1.3 GeV. In fact, a pole around 700 MeV with a width near 600 MeV is found and identified with the κ resonance in Refs [28, 35]. Note, however, that the presence of the $\eta'K$ channel, although being important for the reproduction of the data around 1.3 GeV, is not essential for the understanding of the properties and nature of the κ resonance [40, 42, 43].

Contrary to the weakness of the η' interaction with other pseudoscalars, the S-wave interaction of systems like $K\bar{K}$ is known to be strong, and generates poles related to the $f_0(980)$ and $a_0(980)$ resonances [40, 42, 43]. It is then plausible that in a system like $\eta'K\bar{K}$ the strong attraction in the $K\bar{K}$ system could be enough, together with a weak interaction in the subsystems having a η' , to generate a state with a three-body nature. Such a plausibility should not be surprising because the three-body dynamics is more complex and richer than the one associated with a two-body system, and states of three-body nature can be found even when the interaction in some subsystems is repulsive. Sometimes it is possible to generate a three-body state even when the interaction in all the subsystems is not strong enough to form individual two-body bound states or resonances. Such states are called as borromean states [41]. Thus, the interaction between one or two subsystems can be repulsive or weak, however if the dynamics involved in the remanent subsystem(s) is strong enough to overcome the repulsion/weak attraction, a state of a three-body nature can be formed. This is, indeed, the case of the $KK\bar{K}$, $\phi K\bar{K}$, $J/\psi K\bar{K}$ systems and three-body bound states or resonances are found and associated with the $K(1460)$, $\phi(2170)$ and $Y(4260)$ states, respectively [46–48].

The possibility of finding a three-body state in the $\eta'K\bar{K}$ system has actually been studied earlier in Refs. [51, 52], but

conclusions opposite to each other have been found. While in Ref. [51], when the $\eta'K\bar{K}$ system rearranges as a η' and the $f_0(980)$ resonance, a state is found at 1835 MeV with a width of 70 MeV, no signal of such a state is found in Ref. [52]. The main difference between the two works is the way of dealing with the three-body dynamics. In Ref. [51], for studying the interaction between η' and $f_0(980)$, loops involving these two mesons are introduced and regularized using the dimensional regularization scheme. This implies the introduction of a subtraction constant in the loop function related to the propagation of a meson (η') and a resonance [$f_0(980)$]. In Ref. [52], the formation of states in the $\eta'K\bar{K}$ system is studied within the Faddeev equations in the fixed center approximation approach. In this case, it is assumed that when the η' meson interacts with the $K\bar{K}$ system, which is considered to cluster as the $f_0(980)$ resonance, no changes are produced on the latter. The description of the dynamics in the cluster is introduced through a form factor which depends on the mass and width of the cluster [53].

In this paper, we study the $\eta'K\bar{K}$ system by solving the Faddeev equations with the purpose of looking for possible bound states or/and resonances. We do not assume any cluster formation which cannot be excited in the intermediate scattering states. Such contribution can be important, as noted in Ref. [54]. We obtain the scattering T -matrix of the three-body system as a sum of the Faddeev partitions [55], T_i , such that

$$T = \sum_{i=1}^3 T^i. \quad (4)$$

The formalism used here was developed in Refs. [46–48]. As shown in these latter works, the T_i partitions can be rewritten as

$$T^i = t^i \delta^3(\vec{k}'_i - \vec{k}_i) + \sum_{j \neq i=1}^3 T_R^{ij}, \quad (5)$$

where T_R^{ij} satisfy the equations

$$T_R^{ij} = t^i g^{ij} t^j + t^i \left[G^{ijj} T_R^{ji} + G^{ijk} T_R^{jk} \right], \quad (6)$$

for $i \neq j, j \neq k = 1, 2, 3$. In Eq. (6), the function g^{ij} is the three-body Green's function of the system, which is defined as

$$g^{ij}(\vec{k}'_i, \vec{k}_j) = \left(\frac{N_k}{2E_k(\vec{k}'_i + \vec{k}_j)} \right) \times \frac{1}{\sqrt{s} - E_i(\vec{k}'_i) - E_j(\vec{k}_j) - E_k(\vec{k}'_i + \vec{k}_j) + i\epsilon}, \quad (7)$$

where \sqrt{s} is the energy in the center of mass of the system, the coefficient N_k is equal to 1 for mesons and E_l ($l = 1, 2, 3$) is the energy for the particle l .

The G^{ijk} function in Eq. (6) represents a loop function of three-particles and it is written as

$$G^{ijk} = \int \frac{d^3 k''}{(2\pi)^3} \tilde{g}^{ij} \cdot F^{ijk}, \quad (8)$$

with the elements of \tilde{g}^{ij} being

$$\tilde{g}^{ij}(\vec{k}'', s_{lm}) = \frac{N_l}{2E_l(\vec{k}'')} \frac{N_m}{2E_m(\vec{k}'')} \times \frac{1}{\sqrt{s_{lm}} - E_l(\vec{k}'') - E_m(\vec{k}'') + i\epsilon}, \quad (9)$$

for $i \neq l \neq m$, and the F^{ijk} function, with explicit variable dependence, is given by

$$F^{ijk}(\vec{k}'', \vec{k}'_j, \vec{k}_k, s_{ru}^{k''}) = t^j(s_{ru}^{k''}) g^{jk}(\vec{k}'', \vec{k}_k) \left[g^{jk}(\vec{k}'_j, \vec{k}_k) \right]^{-1} \left[t^j(s_{ru}) \right]^{-1}, \quad (10)$$

for $j \neq r \neq u = 1, 2, 3$. In Eq. (9), $\sqrt{s_{lm}}$ is the invariant mass of the (lm) pair and it depends on the external variables. The upper index k'' in the invariant mass $s_{ru}^{k''}$ of Eq. (10) indicates its dependence on the loop variable (see Refs. [46–50] for more details).

The input two-body t -matrices of Eq. (6) are obtained by solving the Bethe-Salpeter equation in a coupled channel approach

$$t = V + VGt, \quad (11)$$

$$= V + \int \frac{d^4 k}{(2\pi)^4} V \frac{1}{[(P-k)^2 - m_1^2 + i\epsilon][k^2 - m_2^2 + i\epsilon]} t,$$

where the kernel V is determined from the Lagrangian given by Eq. (3). The G function in Eq. (11) stands for the two-body loop function, P and k are, respectively, the total four momentum of the two body system and that of the particles in the loop (expressed in the two-body center of mass frame), and m_1 and m_2 the masses of the two particles under consideration.

The first step of our formalism is to solve Eq. (11) for all the two-body subsystems by considering all the relevant coupled channels into account. In this way, the resonances generated in the two-body subsystems are automatically present in the three-body scattering.

As shown in Refs. [42–45], it is possible to convert the integral Bethe-Salpeter equation [Eq. (11)] into algebraic equations. In this case, the kernel V , and thus, t , can be factorized outside the integral and Eq. (11) becomes

$$t = [1 - VG]^{-1} V, \quad (12)$$

where the loop function G is regularized using dimensional regularization or a cut-off [42, 43].

In a similar fashion, as shown in Refs. [46, 50], equation (6) is also an algebraic set of six coupled equations. This simplification is a result of the cancellation of the contribution of the

off-shell parts of the two-body t -matrices in the three-body Faddeev partitions with the contact term(s) of same topology (whose origin relies in the Lagrangian used to describe the two-body interaction in the subsystems) [46–50]. Interestingly, a deduction of cancellations of two-body and three-body forces using a different procedure has recently been reported in Ref. [56]. Due to these cancellations, only the on-shell part of the two-body t -matrices is relevant to solve Eq. (6). As a consequence, the T_R^{ij} partitions given in Eq. (6) depend only on the total three-body energy, \sqrt{s} , and on the invariant mass of one of the subsystems, which we choose to be the one related to particles 2 and 3 and denote the invariant mass as $\sqrt{s_{23}}$. The other invariant masses, $\sqrt{s_{12}}$ and $\sqrt{s_{31}}$ can be obtained in terms of \sqrt{s} and $\sqrt{s_{23}}$, as shown in Refs. [47, 48].

Using this formalism, we solve Eq. (6) for the $\eta' K \bar{K}$ system. The input two-body $\eta' K$ and $\eta' \bar{K}$ amplitudes are obtained following Ref. [35], where Eq. (11) is solved for the πK , ηK and $\eta' K$ system and, as a result of this coupled channel dynamics, the κ resonance is generated. A good reproduction of the πK phase shift is found up to energies slightly above 1.3 GeV. For the $K \bar{K}$ t -matrix we consider the work of Ref. [42], in which the $\pi\pi$, $K \bar{K}$ system is investigated for the isospin 0 configuration and, for the isospin 1 case, the $K \bar{K}$ and $\pi\eta$ system is considered. Due to the dynamics involved in these coupled channel systems, $f_0(600)$ and $f_0(980)$ are found for the isospin 0 case and $a_0(980)$ for the isospin 1 case. The experimental $\pi\pi$ phase-shifts are well reproduced up to energies around 1.2 GeV.

In Fig. 1 we show the plots obtained for the $\eta' K \bar{K}$ T -matrix for total isospin 0 (left panel) and 1 (right panel) as a function of \sqrt{s} and $\sqrt{s_{23}}$. As can be seen, apart from the threshold enhancement at $(\sqrt{s}, \sqrt{s_{23}}) = (1960, 992)$ MeV in both isospins, no other structure is found. Not even for values of $\sqrt{s_{23}}$ around 980 MeV, where the $K \bar{K}$ system in isospin 0 forms $f_0(980)$ and in isospin 1 forms $a_0(980)$. A threshold enhancement was also the only effect seen in the study of Ref. [52]. At this point a question might arise about the stability of our results when the subtraction constants/cut-offs of the loop functions are varied. In the case of the calculation of the two-body t -matrices, the subtraction constants/cut-offs used here, following Refs. [35] and [42], have been fixed to reproduce relevant data on phase-shifts and inelasticities. We have not varied them due to the limited availability of freedom. For the three-body loop functions, Eq. (8), a cut-off of 1000 MeV has been used. We have varied this cut-off in the range 800-1100 MeV, and minor changes in the size of the three-body amplitudes of Fig. 1 are observed. This insensitivity is related to the presence of three-meson propagators in Eq. (8). Thus, our study of the $\eta' K \bar{K}$ system reveals no structure neither at 1835 MeV, contrary to the finding of Ref. [51], nor above the threshold. Hence, we cannot relate $X(1835)$ and $X(2120)$ with states generated by three-body dynamics. The third X found in Ref. [1], $X(2370)$, is anyways too heavy to be explained as $\eta' K \bar{K}$ resonance. Apart from $X(1835)$, $X(2120)$, there are some π, η states listed in the PDG at energies 1800-2100 MeV with large widths, 100-200 MeV: $\eta(1760)$, $\pi(1800)$, $\eta(2225)$. According to the study

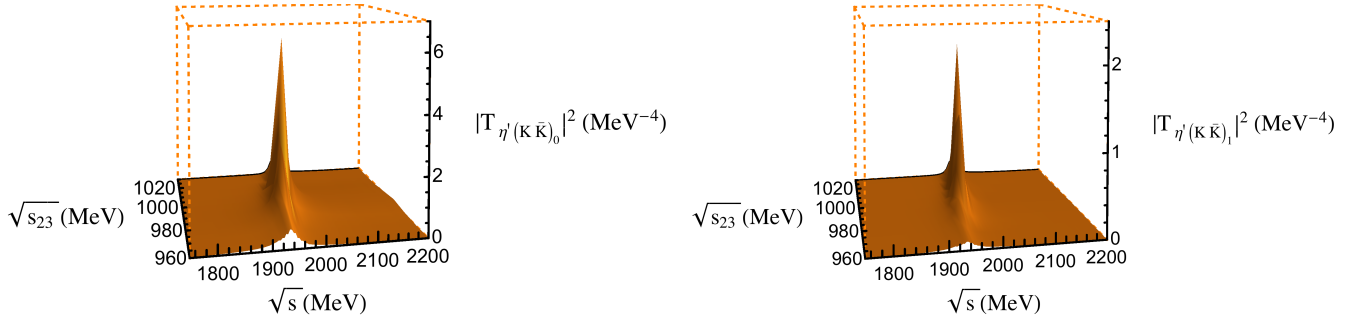


FIG. 1. Modulus squared of the three-body T -matrix for the $\eta' K \bar{K}$ system for total isospin 0, thus the $K \bar{K}$ subsystem is in isospin 0 (left panel) and for total isospin 1, which implies that the $K \bar{K}$ subsystem is in isospin 1 (right panel). The peak seen in both figures corresponds to the three-body threshold cusp.

carried in this work, the dynamics involved in the $\eta' K \bar{K}$ system plays no essential role in understanding the nature of the above mentioned states. We thus conclude from our work that the origin of $X(1835)$ and $X(2120)$ must be something other

than three-pseudoscalar dynamics.

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